Fuzzy Hungarian Method for Solving Intuitionistic Fuzzy Assignment Problems

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ABSTRACT- In this paper we propose a new approach for an intuitionistic fuzzy optimal solution of assignment problems whose decision parameters are triangular intuitionistic fuzzy numbers. We develop a fuzzy version of Hungarian algorithm for the solution of intuitionistic fuzzy assignment problems involving triangular intuitionistic fuzzy numbers without converting them to classical assignment problems. The proposed method is easy to understand and to apply for finding solution of intuitionistic fuzzy assignment problems occurring in real life situations. To illustrate the proposed method numerical examples are provided and the obtained results are discussed.

Index Terms: Intuitionistic Fuzzy numbers, Triangular Intuitionistic Fuzzy numbers, Intuitionistic Fuzzy Assignment Problem, Fuzzy Hungarian Method.

1 Introduction

An assignment problem is a special type of linear programming problem which deals with assigning various activities (jobs or tasks or sources) to an equal number of service facilities (men, machine, laborers etc) on one to one basis in such a way so that the total time or total cost involved is minimized and total sale or total profit is maximized or the total satisfaction of the group is maximized. It is well known that Assignment problems play major role in various areas such as science, engineering and technology, social sciences and many others. In order to solve an assignment problem, the decision parameters of the model must be fixed at crisp values. But to model real-life problems and to perform computations we must deal with uncertainty and inexactness. These uncertainty and inexactness are due to measurement inaccuracy, simplification of physical models, variations of the parameters of the system, computational errors etc. Consequently, we cannot successfully use traditional classical assignment problems and hence the use of fuzzy assignment problems is more appropriate.

The concept of Intuitionistic Fuzzy Set can be viewed as an appropriate/alternative approach to define a fuzzy set in case where available information is not sufficient for the definition of an imprecise concept by means of a conventional fuzzy set. In fuzzy sets the degree of acceptance is considered only but Intuitionistic Fuzzy Set is characterized by a membership function and a non-membership function so that the sum of both values is less than one.

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In 1965 Zadeh[21] introduced the concept of fuzzy sets to deal with imprecision, vagueness in real life situations. In 1970 Bellman and Zadeh[6] proposed the concept of decision making under fuzzy environments. Since then, tremendous development of numerous methodologies and their applications to various decision problems under fuzzy environment have been proposed.

Assignment problems with fuzzy parameters have been studied by several authors, such as Balinski [4] and Chi-Jen Lin[8] and Chen[7], Kuhn Liu and Gao[13] , Sathi Mukherjee and Kajla Basu[17] etc. The intuitionistic fuzzy sets were first introduced by K. Atanassov [3] which is a generalization of the concept of fuzzy set. The intuitionistic fuzzy set is found to be highly useful to deal with vagueness and hence received much attention since its appearance. Senthil Kumar and Jahir Hussain[19] obtained an optimal assignment schedule for a balanced Intuitionistic fuzzy assignment problem involving Intuitionistic triangular fuzzy numbers by using the proposed fuzzy Hungarian method. Jahir Hussian[12] et. al presented an optimal more-for-less solution of mixed constrains intuitionistic fuzzy transportation problems. Mukherjee and Basu[18] proposed an algorithm to solve Intuitionistic Fuzzy Assignment Problem by using Similarity Measures and Score Functions.

Gaurav Kumar, Rakesh Kumar Bajaj [9] have proposed two algorithms-one based on degree of similarity measures and another based on the score function to get the optimal assignment for the Interval-valued Intuitionistic Fuzzy Assignment Problem. Shiny Jose and Sunny Kuriakose[20] proposed an algorithm for solving an Assignment model in

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Intuitionistic fuzzy context.Here we investigate a more realistic problem, namely intuitionistic fuzzy assignment problem based on the assumption that one machine can be assigned exactly one job, also each machine can do at most one job.

The rest of this paper is organized as follows: In section 2, we recall the definition of a new type of arithmetic operations, a new ranking method on Intuitionistic fuzzy numbers and some related results. In section 3, we define Intuitionistic fuzzy assignment problem as an extension of the classical assignment problem and propose fuzzy Hungarian algorithm. In section 4, numerical examples are provided and the obtained results are discussed.

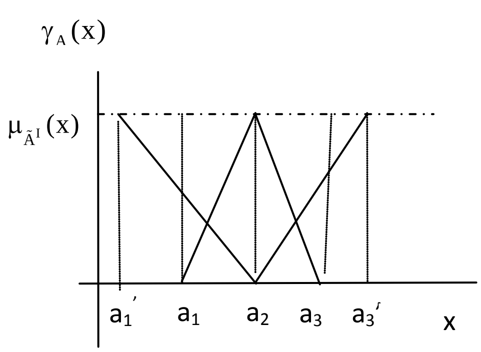
2. PRELIMINARIES

The aim of this section is to present some notations, notions and results which are of useful in our further study.

Definition 2.1. Let X be a universe of discourse, then an Intuitionistic Fuzzy Set (IFS) A in X is given by $\tilde{A}^I = \{ (x, \mu_A(x), \gamma_A(x)) / x \in X \}$ where the functions $\mu_A(x): X \to [0,1]$ and $\gamma_A(x): X \to [0,1]$ determine the degree of membership and degree of non membership of the element $x \in X$, respectively, and for every $x \in X$, $0 \le \mu_A(x) + \gamma_A(x) \le 1$.

Note: Throughout this paper μ represents membership values and γ represents non membership values.

Definition 2.2. For every common fuzzy subset A on X, Intuitionistic Fuzzy Index of x in A is defined as $\pi_A(x) = 1 - \mu_A(x) - \gamma_A(x)$. It is also known as degree of hesitancy or degree of uncertainty of the element x in A. Obviously, for every $x \in X, 0 \le \pi_A(x) \le 1$.



Figuer 1: Membership and non-membership functions of Triangular intuitionistic fuzzy number

Definition 2.3. An Intuitionistic Fuzzy Number (IFN) $\tilde{A}^I$ is an Intuitionistic fuzzy subset of the real line, normal, that is there is any $x_0 \in R$, such that $\mu_{\tilde{A}^I}(x_0) = 1$, $\gamma_{\tilde{A}^I}(x_0) = 0$.

convex for the membership function $\mu_{\tilde{A}^I}(x)$,

i.e., $\mu_{\tilde{A}^I}(\lambda x_1 + (1-\lambda)x_2) \ge \min(\mu_{\tilde{A}^I}(x_1), \mu_{\tilde{A}^I}(x_2))$ for every $x_1, x_2 \in R$, $\lambda \in [0,1]$ is concave for the non-membership $\gamma_{\tilde{A}^I}(x)$, i.e. $\gamma_{\tilde{A}^I}(\lambda x_1 + (1-\lambda)x_2) \le \max(\gamma_{\tilde{A}^I}(x_1), \gamma_{\tilde{A}^I}(x_2))$ for every $x_1, x_2 \in R, \lambda \in [0,1]$.

Definition 2.4. $\tilde{A}^I$ is Triangular Intuitionistic Fuzzy Number (TIFN) with parameters $a'_1 \le a_1 \le a_2 \le a_3 \le a'_3$ and denoted by $\tilde{A}^I = (a_1, a_2, a_3; a'_1, a_2, a'_3)$ having the membership function and non-membership function as follows

$$\mu_{\tilde{A}^I}(x) = \begin{cases} 0 & \text{for } x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1} & \text{for } a_1 \le x \le a_2 \\ 1 & \text{for } x = a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & \text{for } a_2 \le x \le a_3 \\ 0 & \text{for } x > a_3 \end{cases}$$

and

$$\gamma_{\tilde{A}^I}(x) = \begin{cases} 1 & \text{for } x < a'_1 \\ \dfrac{a_2 - x}{a_2 - a'_1} & \text{for } a'_1 \le x \le a_2 \\ 0 & \text{for } x = a_2 \\ \dfrac{x - a_2}{a'_3 - a_2} & \text{for } a_2 \le x \le a'_3 \\ 1 & \text{for } x > a'_3 \end{cases}$$

Note 2.1. Here $\mu_{\tilde{A}^I}(x)$ increases with constant rate for $x \in [a_1, a_2]$ and decreases with constant rate for $x \in [a_2, a_3]$, but $\gamma_{\tilde{A}^I}(x)$ decreases with constant rate for $x \in [a'_1, a_2]$ and increases with constant rate for $x \in [a_2, a'_3]$.

Particular Case:

Let $\tilde{a}^I = (a_1, a_2, a_3; a'_1, a_2, a'_3)$ be a Triangular Intuitionistic fuzzy number then the following cases arises

Case:1 If $a'_1 = a_1$, $a'_3 = a_3$ then $\tilde{A}^I$ represent Triangular Fuzzy number(TFN).

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Case:2 If $a_1^{'} = a_1 = a_3^{'} = a_3 = m$ then $\tilde{A}^I$ represent a real number m.

We denote this triangular Intuitionistic fuzzy number by $\tilde{a}^I = (a_1, a_2, a_3; a_1^{'}, a_2, a_3^{'})$. We use $F(R)$ to denote the set of all Triangular Intuitionistic Fuzzy Numbers.

Also if $m = a_2$ represents the modal value (or) midpoint, $\alpha_1 = (a_2 - a_1)$ represents the left spread and $\beta_1 = (a_3 - a_2)$ right spread of membership function and $\alpha_2 = (a_2 - a_1')$ represents the left spread and $\beta_2 = (a_3^{'} - a_2)$ right spread of non- membership function.

Definition 2.5. Triangular Intuitionistic fuzzy number $\tilde{a}^I \in F(R)$ can also be represented as a pair $\tilde{a}^I = \left(\underline{a}, \overline{a}; \underline{a}^{'}, \overline{a}^{'}\right)$ of functions $\underline{a}(r), \overline{a}(r), \underline{a}^{'}(r)$ and $\overline{a}^{'}(r)$ for $0 \le r \le 1$ which satisfies the following requirements:

$\underline{a}(r)$ is a bounded monotonic increasing left continuous function for membership function.
$\overline{a}(r)$ is a bounded monotonic decreasing left continuous function for membership function.
$\underline{a}(r) \le \overline{a}(r)$, $0 \le r \le 1$
$\underline{a}(r)$ is a bounded monotonic decreasing left continuous function for non- membership function.
$\overline{a}(r)$ is a bounded monotonic increasing left continuous function for non-membership function, $\underline{a}^{'}(r) \le \overline{a}^{'}(r)$, $0 \le r \le 1$

Definition: 2.6. For an arbitrary Triangular Intuitionistic Fuzzy Number $\tilde{a}^I = \left(\underline{a}, \overline{a}; \underline{a}^{'}, \overline{a}^{'}\right)$, the number

$a_0 = \left(\frac{\underline{a}(1) + \overline{a}(1)}{2}\right)$ or $a_0 = \left(\frac{\underline{a}^{'}(1) + \overline{a}^{'}(1)}{2}\right)$ are said to be a

location index number of membership and non-membership functions. The non-decreasing left continuous functions $a_* = (a_0 - \underline{a})$, $a^* = (\overline{a} - a_0)$ are called the left fuzziness index function and the right fuzziness index function for membership function and the non-decreasing left continuous functions $a_*^{'} = (a_0 - \underline{a}^{'})$, $a^{'*} = (\overline{a}^{'} - a_0)$ are called the left fuzziness index function and the right fuzziness index function for non-membership function

$$Mag(\tilde{a}^I) = \frac{1}{2}\left(\int_0^1 (\underline{a} + \overline{a} + 2a_0 + \underline{a}^{'} + \overline{a}^{'})\ f(r)\ dr\right)$$
$$= \frac{1}{2}\left(\int_0^1 (a^* + a^{*'} + 8a_0 - a_* - a_*^{'})\ f(r)\ dr\right).$$

respectively. Hence every triangular Intuitionistic fuzzy number $\tilde{a}^I = (a_1, a_2, a_3; a_1^{'}, a_2, a_3^{'})$ can also be represented by $\tilde{a}^I = \left(a_0, a_*, a^*; a_0, a_*^{'}, a^{'*}\right)$.

2.1. Arithmetic operation on triangular intuitionistic fuzzy numbers:

Ming Ma et al. [16] have proposed a new fuzzy arithmetic based upon both location index and fuzziness index functions. The location index number is taken in the ordinary arithmetic, whereas the fuzziness index functions are considered to follow the lattice rule which is least upper bound in the lattice L. That is for a,b∈L we define $a \vee b = \max\{a, b\}$ and $a \wedge b = \min\{a, b\}$.

For arbitrary triangular intuitionistic fuzzy numbers $\tilde{a}^I = \left(a_0, a_*, a^*; a_0, a_*^{'}, a^{'*}\right)$ and $\tilde{b}^I = \left(b_0, b_*, b^*; b_0, b_*^{'}, b^{'*}\right)$ and $* = \{+, -, \times, \div\}$, the arithmetic operations on the triangular intuitionistic fuzzy numbers are defined by $\tilde{a}^I * \tilde{b}^I = \left(a_0 * b_0,\ a_* \vee b_*,\ a^* \vee b^*; a_0 * b_0,\ a_*^{'} \vee b_*^{'},\ a^{'*} \vee b^{'*}\right)$.

In particular for any two triangular intuitionistic fuzzy numbers $\tilde{a}^I = \left(a_0, a_*, a^*; a_0, a_*^{'}, a^{'*}\right)$ and $\tilde{b}^I = \left(b_0, b_*, b^*; b_0, b_*^{'}, b^{'*}\right)$, we define

Addition:

$$\tilde{a}^I + \tilde{b}^I = \left(a_0, a_*, a^*; a_0, a_*^{'}, a^{'*}\right) + \left(b_0, b_*, b^*; b_0, b_*^{'}, b^{'*}\right)$$
$$= \left(a_0 + b_0, \max\{a_*, b_*\}, \max\{a^*, b^*\}; a_0 + b_0, \max\{a_*^{'}, b_*^{'}\}, \max\{b^{'*}, b^{'*}\}\right)$$

Subtraction:

$$\tilde{a}^I - \tilde{b}^I = \left(a_0, a_*, a^*; a_0, a_*^{'}, a_*^{'}\right) - \left(b_0, b_*, b^*; b_0, b_*^{'}, b^{'*}\right)$$
$$= \begin{pmatrix} a_0 - b_0,\ \max\{a_*, b_*\},\ \max\{a^*, b^*\};\ a_0 - b_0, \\ \max\{a_*^{'}, b_*^{'}\},\ \max\{b^{'*}, b^{'*}\} \end{pmatrix}$$

2.2 Ranking of triangular intuitionistic fuzzy number

Many different approaches for the ranking of intuitionistic fuzzy numbers have been proposed in the literature. Abbasbandy and Hajjari[1] proposed a new ranking method based on the left and the right spreads at some α -levels of fuzzy numbers

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For an arbitrary triangular intuitionistic fuzzy number $\tilde{a}^I = \left(a_0, a_*, a^*; a_0, a'_*, a'^*\right)$ with parametric form $\tilde{a}^I = \left(\underline{a}, \overline{a}; \underline{a}', \overline{a}'\right)$, we define the magnitude of the triangular intuitionistic fuzzy number $\tilde{a}^I$ by

where the function $f(r)$ is a non-negative and increasing function on $[0,1]$ with $f(0)=0$, $f(1)=1$ and $\int_0^1 f(r)\,dr = \frac{1}{2}$. The function $f(r)$ can be considered as a weighting function. In real life applications, $f(r)$ can be chosen by the decision maker according to the situation. In this paper, for convenience we use $f(r)$.

Hence

$$Mag(\tilde{a}^I) = \left(\frac{a^* + a^{*'} + 8a_0 - a_* - a'_*}{4}\right) = \left(\frac{\underline{a} + \overline{a} + 2a_0 + \underline{a}' + \overline{a}'}{4}\right).$$

The magnitude of a triangular intuitionistic fuzzy number $\tilde{a}^I$ synthetically reflects the information on every membership degree, and meaning of this magnitude is visual and natural. $Mag(\tilde{a}^I)$ is used to rank intuitionistic fuzzy numbers. The larger $Mag(\tilde{a}^I)$ is larger intuitionistic fuzzy number.

For any two triangular intuitionistic fuzzy numbers $\tilde{a}^I = \left(a_0, a_*, a^*; a_0, a'_*, a'^*\right)$ and $\tilde{b}^I = \left(b_0, b_*, b^*; b_0, b'_*, b'^*\right)$ in F(R), we define the ranking of $\tilde{a}^I$ and $\tilde{b}^I$ by comparing the Mag ($\tilde{a}^I$) and Mag ($\tilde{b}^I$) on R as follows:

$\tilde{a}^I \succeq \tilde{b}^I$ if and only if $Mag(\tilde{a}^I) \geq Mag(\tilde{b}^I)$

$\tilde{a}^I \preceq \tilde{b}^I$ if and only if $Mag(\tilde{a}^I) \leq Mag(\tilde{b}^I)$

$\tilde{a}^I \approx \tilde{b}^I$ if and only if $Mag(\tilde{a}^I) = Mag(\tilde{b}^I)$

3. MAIN RESULTS

Suppose there are n activities (jobs or tasks or sources) to be performed and n service facilities (men, machine, laborers etc) are available for doing these activities. Assume that each service facility can perform one activity at a time. The objective of the problem is to assign these activities to the service facilities on one to one basis in such a way so that the total time or total cost involved is minimized and total sale or total profit is maximized or the total satisfaction of the group is maximized.

3.1 Mathematical Model of Intuitionistic Fuzzy Assignment Problem

Let the ith person is assigned to the jth job and is denoted by $\tilde{x}_{ij}$ and $\tilde{c}^I_{ij}$ be the corresponding Intuitionistic fuzzy cost of assigning the ith person to the jth job. Since the objective is to minimize the overall Intuitionistic fuzzy cost for performing all jobs, the mathematical model of this Intuitionistic fuzzy assignment problem is as follows:

$$\text{Minimize } \tilde{z}^I = \sum_{i=1}^{n}\sum_{j=1}^{n} \tilde{c}^I_{ij}\tilde{x}_{ij}$$

$$\text{subject to } \sum_{i=1}^{n} \tilde{x}_{ij} \approx \tilde{1}, \quad j = 1, 2, ..., n \qquad (3.1)$$

$$\sum_{j=1}^{n} \tilde{x}_{ij} \approx \tilde{1}, \quad i = 1, 2, ..., n$$

$$\tilde{x}_{ij} \approx \tilde{0} \text{ or } \tilde{1}, \quad j = 1, 2, ..., n$$

$$\text{where } \tilde{x}_{ij} \approx \begin{cases} \tilde{1}, & \text{if the } i^{th} \text{person is assigned to } j^{th} \text{ job} \\ \tilde{0}, & \text{if the } i^{th} \text{person is not assigned to } j^{th} \text{ job} \end{cases}$$

This Intuitionistic fuzzy Assignment problem can be stated in the form of $(n \times n)$ fuzzy cost matrix $\tilde{c}^I_{ij}$ be the corresponding Intuitionistic fuzzy cost of assigning the ith person to the jth job as given in the following table:

Table 3. 1: Intuitionistic Fuzzy cost matrix of Intuitionistic fuzzy assignment problem

	Jobs							
Persons		1	2	3	…	j	…	n
	1	$\tilde{c}^I_{11}$	$\tilde{c}^I_{12}$	$\tilde{c}^I_{13}$	…	$\tilde{c}^I_{1j}$	…	$\tilde{c}^I_{1n}$
	2	$\tilde{c}^I_{21}$	$\tilde{c}^I_{22}$	$\tilde{c}^I_{23}$	…	$\tilde{c}^I_{2j}$	…	$\tilde{c}^I_{2n}$
	…	…	…	…	…	…	…	…
	i	$\tilde{c}^I_{i1}$	$\tilde{c}^I_{i2}$	$\tilde{c}^I_{i3}$	…	$\tilde{c}^I_{ij}$	…	$\tilde{c}^I_{in}$
	…	…	…	…	…	…	…	…
	n	$\tilde{c}^I_{n1}$	$\tilde{c}^I_{n2}$	$\tilde{c}^I_{n3}$	…	$\tilde{c}^I_{nj}$	…	$\tilde{c}^I_{nn}$

3.2 Fuzzy Hungarian method

We, now introduce a new algorithm called the Intuitionistic fuzzy Hungarian method for finding a Intuitionistic fuzzy optimal assignment for Intuitionistic fuzzy assignment problem.

Step 1: Determine the Intuitionistic fuzzy cost table from the given problem. If the number of sources is equal to the number of destinations go to step 3. If the number of sources is not equal to the number of destinations go to step 2.

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Step 2: Add a dummy source or dummy destination, so that the Intuitionistic fuzzy cost table becomes a Intuitionistic fuzzy square matrix. The Intuitionistic fuzzy cost entries of dummy source/destinations are always Intuitionistic fuzzy zero.

Step 3: Subtract the row minimum from each row entry of that row.

Step 4: Subtract the column minimum of the resulting Intuitionistic fuzzy Assignment problem after using step 3 from each column entry of that column.

Each column and row now has at least one fuzzy zero.

Step 5: In the modified Intuitionistic fuzzy assignment table obtained in step 4, search for Intuitionistic fuzzy optimal assignment as follows.

Examine the rows successively until a row with a single Intuitionistic fuzzy zero is found. Assign the Intuitionistic fuzzy zero and cross off all other Intuitionistic fuzzy zeros in its column. Continue this for all the rows.

Repeat the procedure for each column of reduced Intuitionistic fuzzy assignment table.

If a row and / or column have two or more Intuitionistic fuzzy zeros assign arbitrary any one of these Intuitionistic fuzzy zeros and cross off all other Intuitionistic fuzzy zeros of that row/column. Repeat (a) through (c) above successively until the chain of assigning or cross ends.

Step 6: If the number of assignments is equal to n, the order of the Intuitionistic fuzzy cost matrix, Intuitionistic fuzzy optimal solution is reached. If the number of assignments is less than n, the order of the Intuitionistic fuzzy zeros of the Intuitionistic fuzzy cost matrix, go to the step 7.

Step 7: Draw the minimum number of horizontal and / or vertical lines to cover all the Intuitionistic fuzzy zeros of the reduced Intuitionistic fuzzy assignment matrix. This can be done by using the following:

Mark rows that do not have any assigned Intuitionistic fuzzy zero.

Mark columns that have Intuitionistic fuzzy zeros in the marked rows.

Mark rows that do have Intuitionistic assigned fuzzy zeros in the marked columns.

Repeat ii) and iii) above until the chain of marking is completed. Draw lines through all the unmarked rows and marked columns. This gives the desired minimum number of lines.

Step 8: Develop the new revised reduced Intuitionistic fuzzy cost matrix as follows:

Find the smallest entry of the reduced fuzzy Intuitionistic cost matrix not covered by any of the lines. Subtract this entry from all the uncovered entries and add the same to all the entries lying at the intersection of any two lines.

Step 9: Repeat step 6 to step 8 until Intuitionistic fuzzy optimal solution to the given Intuitionistic fuzzy assignment problem is attained.

4. NUMERICAL EXAMPLES

Example 4.1 Consider an intuitionistic fuzzy assignment problem discussed by senthil kumar et al [19] with rows representing three machines M_1, M_2, M_3 and columns representing the three jobs J_1, J_2, J_3. The cost matrix $\left[\tilde{C}_{ij}^{\,I}\right]$ is given whose elements are triangular intuitionistic fuzzy numbers. The problem is to find the optimal assignment so that the total cost of job assignment becomes minimum.

$$\begin{bmatrix} (7,21,29;2,21,34) & (7,20,57;3,20,61) & (12,25,56;8,25,60) \\ (8,9,16;2,9,22) & (4,12,35;1,12,38) & (6,14,28;3,14,31) \\ (5,9,22;2,9,25) & (10,15,20;5,15,25) & (4,16,19;1,16,22) \end{bmatrix}$$

To apply the proposed algorithm and the fuzzy arithmetic, let us express all the triangular intuitionistic fuzzy numbers in the given problem based upon both location index and fuzziness index functions. That is in the form of we have

$$\begin{bmatrix} (21,14-14r,8-8r;21,19-19r,13-13r) & (20,13-13r,37-37r;20,17-17r,41-41r) & (25,13-13r,31-31r;25,17-17r,35-35r) \\ (9,1-1r,7-7r;9,7-7r,13-13r) & (12,8-8r,23-23r;12,11-11r,26-26r) & (14,8-8r,14-14r;14,11-11r,17-17r) \\ (9,4-4r,13-13r;9,7-7r,16-16r) & (15,5-5r,5-5r;15,10-10r,10-10r) & (16,12-12r,3-3r;16,15-15r,6-6r) \end{bmatrix}$$

The given problem is a balanced one. So using step 3 of the Intuitionistic fuzzy Hungarian method we obtain

$$\begin{bmatrix} (1,14-14r,37-37r;1,19-19r,41-41r) & (\tilde{0}) & (5,13-13r,37-37r;5,17-17r,41-41r) \\ (\tilde{0}) & (3,8-8r,23-23r;3,11-11r,26-26r) & (5,8-8r,14-14r;5,11-11r,17-17r) \\ (\tilde{0}) & (6,5-5r,13-13r;6,10-10r,16-16r) & (7,12-12r,13-13r;7,15-15r,16-16r) \end{bmatrix}$$

Using step 4 of the Intuitionistic fuzzy Hungarian method we obtain the following modified Intuitionistic fuzzy

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assignment matrix.

$$\begin{bmatrix} (1,14-14r,37-37r;1,19-19r,41-41r) & (\tilde{0}) & (\tilde{0}) \\ (\tilde{0}) & (3,8-8r,23-23r;3,11-11r,26-26r) & (\tilde{0}) \\ (\tilde{0}) & (6,5-5r,13-13r;6,10-10r,16-16r) & (2,12-12r,14-14r;2,15-15r,17-17r) \end{bmatrix}$$

Now using the step 8 of the Intuitionistic fuzzy Hungarian method and repeating the procedure, we have the

following Intuitionistic fuzzy optimal assignment matrix.

$$\begin{bmatrix} (1,14-14r,37-37r;1,19-19r,41-41r) & [\,(\tilde{0})\,] & (\not{\tilde{0}}) \\ (\not{\tilde{0}}) & (3,8-8r,23-23r;3,11-11r,26-26r) & [\,(\tilde{0})\,] \\ [(\tilde{0})] & (6,5-5r,13-13r;6,10-10r,16-16r) & (2,12-12r,14-14r;2,15-15r,17-17r) \end{bmatrix}$$

Therefore, the Intuitionistic fuzzy optimal assignment for the given Intuitionistic fuzzy assignment problem is

$$M_1 \to J_2, M_2 \to J_3, M_3 \to J_1.$$

The Intuitionistic fuzzy optimal total cost is calculated as $=\left(20,13-13r,37-37r;20,17-17r,41-41r\right)+$
$\left(14,8-8r,14-14r;14,11-11r,17-17r\right)+$
$\left(9,4-4r,13-13r;9,7-7r,16-16r\right)$

$=\left(34,13-13r,37-37r;34,17-17r,41-41r\right)+$
$\left(9,4-4r,13-13r;9,7-7r,16-16r\right)$
$=(43,13-13r,37-37r;43,17-17r,41-41r)$

Intuitionistic fuzzy optimal assignment cost is (if r=0) = (30, 43, 80; 36, 43, 84) units.

Intuitionistic fuzzy optimal assignment $M_1 \to J_2, M_2 \to J_3, M_3 \to J_1.$	
Intuitionistic fuzzy optimal assignment cost is $(43,13-13r,37-37r;43,17-17r,41-41r)$	
If r=0	(30,43,80; 36,43,84)
If r=0.25	(33.25,43,70.75; 30.25,43,73.75)
If r=0.5	(36.5,43,61.5; 34.5,43,63.5)
If r=0.75	(39.75,43,52.25; 38.75,43,53.25)
If r=1	43

$$\begin{bmatrix} (21,14-14r,8-8r;21,19-19r,13-13r) & (20,13-13r,37-37r;20,17-17r,41-41r) & (25,13-13r,31-31r;25,17-17r,35-35r) \\ (9,1-1r,7-7r;9,7-7r,13-13r) & (12,8-8r,23-23r;12,11-11r,26-26r) & (14,8-8r,14-14r;14,11-11r,17-17r) \\ (9,4-4r,13-13r;9,7-7r,16-16r) & (15,5-5r,5-5r;15,10-10r,10-10r) & (16,12-12r,3-3r;16,15-15r,6-6r) \end{bmatrix}$$

$$\begin{bmatrix} (21,14-14r,8-8r;21,19-19r,13-13r) & (20,13-13r,37-37r;20,17-17r,41-41r) & (25,13-13r,31-31r;25,17-17r,35-35r) \\ (9,1-1r,7-7r;9,7-7r,13-13r) & (12,8-8r,23-23r;12,11-11r,26-26r) & (14,8-8r,14-14r;14,11-11r,17-17r) \\ (9,4-4r,13-13r;9,7-7r,16-16r) & (15,5-5r,5-5r;15,10-10r,10-10r) & (16,12-12r,3-3r;16,15-15r,6-6r) \end{bmatrix}$$

Where P.Senthil kumar et al got the the Intuitionistic fuzzy optimal assignment for the given Intuitionistic fuzzy assignment problem is $M_1 \to J_1, M_2 \to J_2, M_3 \to J_3$. and the Intuitionistic fuzzy optimal assignment cost is (15,49,83; 4,49,94)

5. CONCLUSION

We have thus obtained an optimal assignment schedule for a Intuitionistic fuzzy assignment problem using triangular Intuitionistic fuzzy number by the proposed new algorithm. It can be seen that the fuzzy optimal solution to the Intuitionistic assignment problem given in example 4.1. From the results we see that the proposed fuzzy Hungarian method is gives better assignment.

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